INTEGRAL TAK TENTU

Rumus Dasar :

1. $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$
2. $\int x^{-1}dx = \int \frac{1}{x}\,dx = \ln x + C$
3. $\int \cos x\,dx = \sin x + C$
4. $\int \sin x\,dx = -\cos x + C$
5. $\int e^x dx = e^x + C$

Rumus Substitusi

Substitusi : u = f(x) $\rightarrow$ du = f'(x)dx

1. $\int u^n du = \frac{1}{n+1}u^{n+1} + C$
2. $\int u^{-1}du = \int \frac{1}{u}\,du = \ln u + C$
3. $\int \cos u\,du = \sin u + C$
4. $\int \sin u\,du = -\cos u + C$
5. $\int e^u du = e^u + C$

Rumus Perluasan:

1. $\int [f(x)]^n\,df(x) = \int [f(x)]^n f'(x)dx = \frac{1}{n+1}[f(x)]^{n+1} + c$
2. $\int \frac{1}{f(x)}df(x) = \int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$
3. $\int \cos f(x)df(x) = \int [\cos f(x) f'(x)dx = \sin f(x) + C]$
4. $\int \sin f(x)df(x) = \int [\sin f(x) f'(x)dx = -\cos f(x) + C]$
5. $\int e^{f(x)}df(x) = \int e^{f(x)} f'(x)dx = e^{f(x)} + C$

Contoh :

1. $\int (2+x^3)^2.3x^2dx = \int (2+x^3)^2.d(2+x^3) = \frac{1}{3}(2+x^3)^3 + C$

Or :

Subst : u = (2 + x^3) $\rightarrow$ du=$3x^2$

$\rightarrow \int (2+x^3)^2.3x^2dx = \int u^2.du = \frac{1}{3}u^3 + C = \frac{1}{3}(2+x^3)^3 + C$

2. $\int x(1-x^2)^5 dx = \int (1-x^2)^5 d(1-x^2).-\frac{1}{2} = -\frac{1}{2}.\frac{1}{6}(1-x^2)^6 + C$

$$= -\frac{1}{12}(1-x^2)^6 + C$$

or :

Subst : u = 1 – x^2 $\rightarrow$ du = - 2x dx $\rightarrow$ x dx = - ½ du

$\rightarrow \int x(1-x^2)dx = \int u^5.-\frac{1}{2}du = -\frac{1}{2}.\frac{1}{6}u^6 + C = -\frac{1}{12}(1-x^2)^6 + C$

3. $\int \frac{2x}{x^2+1}dx = \int \frac{1}{x^2+1}d(x^2+1) = 1n\left|x^2+1\right| + C$

or :

Subst : u = x^2 + 1 $\rightarrow$ du = 2x dx

$\int \frac{2x}{x^2+1}dx = \int \frac{1}{u}d(u) = 1n|u| = 1n\left|x^2+1\right| + C$

4. $\int \frac{x-1}{x^2-2x+3}dx = \int \frac{1/2.(2x-2)}{x^2-2x+3}dx = \frac{1}{2}1n\left|x^2-2x+3\right| + C$

Or:

Subst : u = x^2 – 2x + 3 $\rightarrow$ du = (2x – 2)dx $\rightarrow$ (x – 1) dx = ½ du

$\rightarrow \int \frac{x-1}{x^2-2x+3}dx = \int \frac{1}{u}.1/2du = \frac{1}{2}1n|u| = \frac{1}{2}1n\left|x^2-2x+3\right| + C$

5. $\int e^{3x+2}dx = \frac{1}{3}\int e^{3x+2}d(3x+2) = \frac{1}{3}.e^{3x+2} + C$

Or:

Subst : u = 3x + 2 $\rightarrow$ du = 3 dx $\rightarrow$ dx = 1/3 du

$\int e^{3x+2}dx = \int e^u.\frac{1}{3}du = \frac{1}{3}.e^u + C = \frac{1}{3}.e^{3x+2} + C$

6. $\int \cos(x^2+1).x.dx = \frac{1}{2}\int \cos(x^2+1).d(x^2+1) = \frac{1}{2}\sin(x^2+1) + C$

Or:

Subst : u = x^2 + 1 $\rightarrow$ du = 2x dx $\rightarrow$ x.dx = 1/2 du

$$\int \cos(x^2+1).x.dx = \int \cos u.\frac{1}{2}du = \frac{1}{2}\sin u + C = \frac{1}{2}\sin(x^2+1) + C$$

7. $\int \sin(1-3x).dx = -\frac{1}{3}\int \sin(1-3x).d(1-3x) = -\frac{1}{3}\left[-\cos(1-3x)\right] + C$

$$= \frac{1}{3}\cos(1-3x) + C$$

Or:

Subst : u = 1 - 3x $\rightarrow$ du = - 3dx $\rightarrow$ dx = - 1/3 du

$$\int \sin(1-3x).dx = \int \sin u.-\frac{1}{3}dx = -\frac{1}{3}\left[-\cos u\right] + C = \frac{1}{3}\cos(1-3x) + C$$

Latihan :

1. $\int \frac{1}{2x}dx$
2. $\int \frac{x^2-x}{x+1}dx$
3. $\int \frac{\sin x}{\cos x}dx$
4. $\int \frac{x-3}{x^2-6x-6}dx$
5. $\int \frac{e^x}{1+3e^x}.dx$
6. $\int e^{3x^2}.2xdx$
7. $\int \sin x.e^{\cos x}dx$
8. $\int \frac{x}{(x^2-1)^3}.dx$
9. $\int \sqrt{2-x}.dx$
10. $\int x^{3/2} + x^{1/2}.dx$

INTEGRAL PARSIAL

Rumus Integral Parsial:

$$\boxed{\int u.dv = uv = \int v.du} \qquad v = \int dv$$

Contoh:

1. $\int x \sin x.dx$

   Subst : u = x $\rightarrow$ du = dx

   dv = sin x dx $\rightarrow$ v = - cos x

   $\rightarrow \int x\sin x.dx = \int x.d(-\cos x) = x.(-\cos x) - \int -\cos x.dx$

   $= -x.\cos x + \sin x + C$

2. $\int 1nx.dx$

Subst : u = 1nx $\rightarrow$ du = 1/x dx

dv = dx $\rightarrow$ v = x

$$\int 1nx.dx = x.1nx - \int x.d(1nx) = x.1n\,x - \int x.\frac{1}{x}dx$$

$$= x.1n\,x - \int .dx = x1n\,x - x + C$$

$$\int x.e^x dx = \int x.d(e^x) = x.e^x - \int .e^x dx = x.e^x - e^x + C$$

3. $\int x.\cos x\, dx$

Subst : u = x $\rightarrow$ du = dx

dv = cos x dx $\rightarrow$ v = sin x

$$\int x.\cos xdx = \int x.d(\sin x) = x.\sin x - \int \sin x + \cos x + C$$

4. $\int x^2.\sin xdx$

Subst : u = x^2 $\rightarrow$ du = 2x . dx

dv = sin x dx $\rightarrow$ v = - cos x

$$\int x^2.\sin xdx = \int x^2.d(-\cos x) = -x^2.\cos x - \int -\cos x.2xdx$$

$$= -x^2.\cos x + 2\int x.\cos xdx \text{ (dari contoh 4)}$$

$$= -x^2.\cos x + 2(x.\sin x + \cos x) + C$$

$$= -x^2.\cos x + 2x.\sin x + 2\cos x + C$$

5. $\int e^x.\sin xdx$

Subst : u = e^x $\rightarrow$ du = e^x . dx

dv = sin x dx $\rightarrow$ v = - cos x

$$\int e^x.\sin xdx = \int e^x.d(-\cos x) = -e^x.\cos x - \int -\cos x.e^x dx$$

$$= -e^x.\cos x + \int e^x.\cos x\, dx$$

$$= -e^x.\cos x + \int e^x.d(\sin x) = -e^x.\cos x + e^x.\sin x - \int e^x.\sin\, dx$$

$$2\int e^x.\sin x\, dx = -e^x.\cos x + e^x.\sin x$$

$$\int e^x.\sin x\, dx = 1/2(-e^x.\cos x + e.\sin x) + C$$

INTEGRAL TERTENTU

<u>Notasi :</u>

$\int_a^b f(x)dx$ → dibaca integral tertentu dari f(x) terhadap x, dari x = a sampai x = b.

<u>Teorema</u> (Teorema Dasar Kalkulus)

Jika f(x) kontinu dalam interval [a, b] dan jika F(x) adalah hasil integral tentu dari f(x), maka :

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$$

Sifat-sifat integral tertentu :

1) $\int_a^a f(x)dx = 0$

2) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

3) $\int_a^b c.f(x)dx = c.\int_a^b f(x)$

4) $\int_a^b [f(x) \pm g(x)dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

5) $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$, dengan a <b <c

<u>Contoh-contoh :</u>

1) $\int_0^5 xdx = \left[\frac{1}{2}x^2\right]_0^5 = 1/2(5)^2 - 1/2(0)^2 = 25/2$

2) $\int_{0}^{5}(x^2+1)dx = \left[\frac{1}{3}x^3 + x\right]_1^2 = [1/3.(2)^3+2]-[1/3.(1)^3+1] = 10/3$

3) $\int_{0}^{\pi} 3.\sin x dx = [-3\cos x]_0^{\pi} = -3(-1)+3(1) = 3+3 = 6$

4) $\int_{-1}^{0}(3x^2-2x+3)dx + \int_{0}^{2}(3x^2-2x++3)dx = \int_{-1}^{2}(3x^2-2x+3)dx$

$= [x^3 - x^2 + 3x]_{-1}^{3} = [27-9+9]-[-1-1-3] = 27+5 = 32$

5) $\int_{0}^{4} x\sqrt{x^2+9dx}$ $\rightarrow$ subst : u x^2 + 9 $\rightarrow$ du = 2x dx

$\rightarrow \int x\sqrt{x^2+9}dx = \int u^{1/2}.1/2du = 1/2(2/3.u^{3/2}) + C = 1/3u^{3/2} + C$

= 1/3 $(x^2 + 9)^{3/2}$ + C

$\rightarrow \int_{0}^{4} x\sqrt{x^2+9dx} = [1/3(x^2+9)^{3/2}]_0^4 = 125/3 - 27/3 = 98/3$

6) $\int_{0}^{1}\sqrt{x^2 x}(2x+1)dx$

subst : u = x2 + x $\rightarrow$ du = (2x + 1) dx

Batas : x = 0 $\rightarrow$ u = 2

x = 1 $\rightarrow$ u = 2

$\rightarrow \int_{0}^{1}\sqrt{x^2+x}(2x+1)dx = \int_{0}^{2}u^{1/2}du = \frac{2}{3}[u^{3/2}]_0^2$

= 2/3 $[2\sqrt{2} - 0]$

= 4/3. $\sqrt{2}$

7) $\int_{0}^{1}\frac{x+1}{(x^2+2x+6)^2}dx$

subst : u = x2 + 2x + 6 $\rightarrow$ du = (x + 1) dx

Batas : x = 0 $\rightarrow$ u = 6

x = 1 $\rightarrow$ u = 9

$\rightarrow \int_{0}^{1}\frac{x+1}{(x^2+2x+6)^2}dx = \int_{6}^{9}u^{-2} = \left[-\frac{1}{2}\frac{1}{u}\right]_6^9 = -\frac{1}{18} - \left(\frac{-1}{12}\right) = \frac{1}{36}$