Kalkulus (1240033)

Pertemuan Ke-9

Turunan

Juwairiah, S.Si,M.T
(juwai_riah@yahoo.com)

Sub Pokok Bahasan

- Turunan Fungsi Implisit
- Turunan Fungsi secara Logaritmis
- Turunan Fungsi Parameter

Kompetensi Khusus

Mahasiswa mampu menyelesaikan berbagai turunan fungsi

TURUNAN FUNGSI IMPLISIT

Fungsi eksplisit : $y = f(x)$

Fungsi implisit : $y - f(x) = 0 \; atau \; F(x,y) = 0$

Kunci Penyelesaian :

1. Semua turunan y selalu diikuti dengan dy/dx
2. Mengikuti aturan turunan

Contoh :

1)Tentukan $\frac{dy}{dx}$ dari persamaan $y^3 + 7y = x^3$

Jawab :

$$\frac{d}{dx}(y^3 + 7y) = \frac{d}{dx}(x^3)$$

$$3y^2\frac{dy}{dx} + 7.\frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx}\left(3y^2 + 7\right) = 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2}{3y^2 + 7}$$

2)Cari $\frac{dy}{dx}$ jika : $x^2 + 5y^3 = x + 9$

Jawab : $\frac{d}{dx}\left(x^2 + 5y^3\right) = \frac{d}{dx}(x+9)$

$$2x + 15y^2\frac{dy}{dx} = 1$$

$$15y^2\frac{dy}{dx} = 1 - 2x$$

$$\frac{dy}{dx} = \frac{1-2x}{15y^2}$$

3)Cari $\frac{dy}{dt}$ jika $t^3 + t^2y - 10y^4 = 0$

Jawab :

$$\frac{d}{dt}(t^3 + t^2 y - 10y^4) = \frac{d}{dt}(0)$$

$$3t^2 + t^2.\frac{dy}{dx} + 2t.y - 40y^3.\frac{dy}{dt} = 0$$

$$\frac{dy}{dt}\left(t^2 - 40y^3\right) = -3t^2 - 2ty$$

$$\frac{dy}{dt} = \frac{3t^2 + 2ty}{40y^3 - t^2}$$

4) Cari $\frac{dy}{dx}$ jika $x^3 + y^3 = 6xy$

Jawab : $\frac{d}{dx}(x^3 + y^3) = \frac{d}{dx}(6xy)$

$$3x^2 + 3y^2 . \frac{dy}{dx} = 6y + 6x \frac{dy}{dx}$$

$$(3y^2 - 6x)\frac{dy}{dx} = 6y - 3x^2$$

$$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$$

TURUNAN FUNGSI SECARA LOGARITMIS

Diketahui : $y = [f(x)]^{g(x)}$, tentukan $\frac{dy}{dx}$!

$$\ln y = ln[f(x)]^{g(x)}$$

$$= g(x).\ln[f(x)]$$

Kedua sisi diturunkan ; $\frac{1}{y}.\frac{dy}{dx} = g'(x).\ln[f(x)] + g(x).\frac{f'(x)}{f(x)}$

$$\frac{dy}{dx} = y[g'(x).lnf(x) + g(x).\frac{f'(x)}{f(x)}]$$

$$= [f(x)]^{g(x)}.[g'(x).lnf(x) + g(x).\frac{f'(x)}{f(x)}]$$

Contoh :

1) $y = (x+1)^x$

$$\frac{dy}{dx} = (x+1)^x[1.\ln(x+1) + x.\frac{1}{x+1}]$$

$$= (x+1)^x[\ln(x+1) + \frac{x}{x+1}]$$

2) $y = (tg\ x)^{\sin x} \rightarrow$

$$\frac{dy}{dx} = (tg\ x)^{\sin x}.[\cos x.\ln(tg\ x) + \sin x.\frac{\sec^2 x}{tg\ x}]$$

$$= (tg\ x)^{\sin x}[\cos x.\ln(tg\ x) + \sec x]$$

TURUNAN FUNGSI PARAMETER

Jika : $y = f(t)$

$x = g(t)$

Maka : $y' = \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ → Turunan ke -1

Dan :

$$y'' = \frac{d^2y}{dx^2} = \frac{d/dt\left(dy/dt\right)}{dx/dt}$$ → Turunan ke-2

Contoh :

1)Diketahui : $y = 2t^2 + 3t \; dan \; x = 1 + 2t$

Maka

$$\frac{dy}{dt} = 4t + 3 \qquad \frac{dx}{dt} = 2$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4t+3}{2} = 2t + \frac{3}{2}$$

$$\frac{d^2y}{dx^2} = \frac{d/d\square\,(dy/dx)}{dx/dt} = \frac{d/dt(2t+\frac{3}{2})}{2} = \frac{2}{2} = 1$$

2) $y = \cos t \; dan \; x = \sin t$

$$\frac{dy}{dt} = -\sin t \qquad \frac{dx}{dt} = \cos t$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-\sin t}{\cos t} = -tg\, t$$

$$\frac{d^2y}{dx^2} = \frac{d/dt\left(dy/dt\right)}{dx/dt} = \frac{d/dt(-tg\, t)}{\cos t}$$

$$= \frac{-sec^2 t}{\cos t}$$

$$= -\, sec^2 t.\sec t \quad = -\, sec^3 t$$

Referensi

- Purcell, Varberg, *Kalkulus dan Geometri Analitis*, Penerbit Erlangga, 1993
- Frank Ayres, *Calculus*, Mc.Graw Hill, New York, 1972
- J.Salas and Hill, *Calculus One and Several Variables*, John Willey& Sons, NewYork, 1982