

FRACTAL GEOMETRY

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Fractal

- *Berasal dari Bahasa Latin: Fractus, frangere: fraction (English): “to break”*
 - To create irregular fragments
 - To fragmented
 - In fraction or refraction
 - In irregular



Geometry

Geometry (from the Ancient Greek: γεωμετρία; *geo-* "earth", *-metron* "measurement") is a branch of mathematics concerned with questions of shape, size, relative position of figures, and the properties of space. A mathematician who works in the field of geometry is called a geometer.



Geometry

Mathematicians divide geometries into two types:

- Euclidean geometry
- Fractal geometry



Dimension

- The **dimension** of a mathematical space (or object) is informally defined as the minimum number of coordinates needed to specify any point within it ([Wikipedia](#))
- Jumlah minimal koordinat yang dibutuhkan untuk menentukan titik-titik yang ada di dalamnya ([Wikipedia](#))



Dimension

- Topology:
 - a magnitude that, independently or in conjunction with other such magnitudes, serves to define the location of an element within a given set, as of a point on a line, an object in a space, or an event in space-time (Suatu besaran yang secara independen atau bersamaan dengan besaran lainnya, berfungsi untuk menentukan lokasi sebuah elemen dalam himpunan tertentu, misalnya sebuah titik pada garis, suatu objek dalam ruang, atau suatu peristiwa dalam ruang-waktu).



Dimension

- Topology:
 - the number of elements in a finite basis of a given vector space. (jumlah elemen dalam basis terbatas dari suatu ruang vektor tertentu).
- Physics: any of a set of basic kinds of quantity, as mass, length, and time, in terms of which all other kinds of quantity can be expressed (suatu himpunan bilangan dasar, misalnya massa, panjang, dan waktu, yang dalam hal mana semua jenis kuantitas lainnya dapat diekspresikan).



Bilangan

- Bilangan bulat: integer values: 0, 1, 2, 3
- Bilangan pecahan: non integer values: 0.5, 1.234, 2.709, 3.05

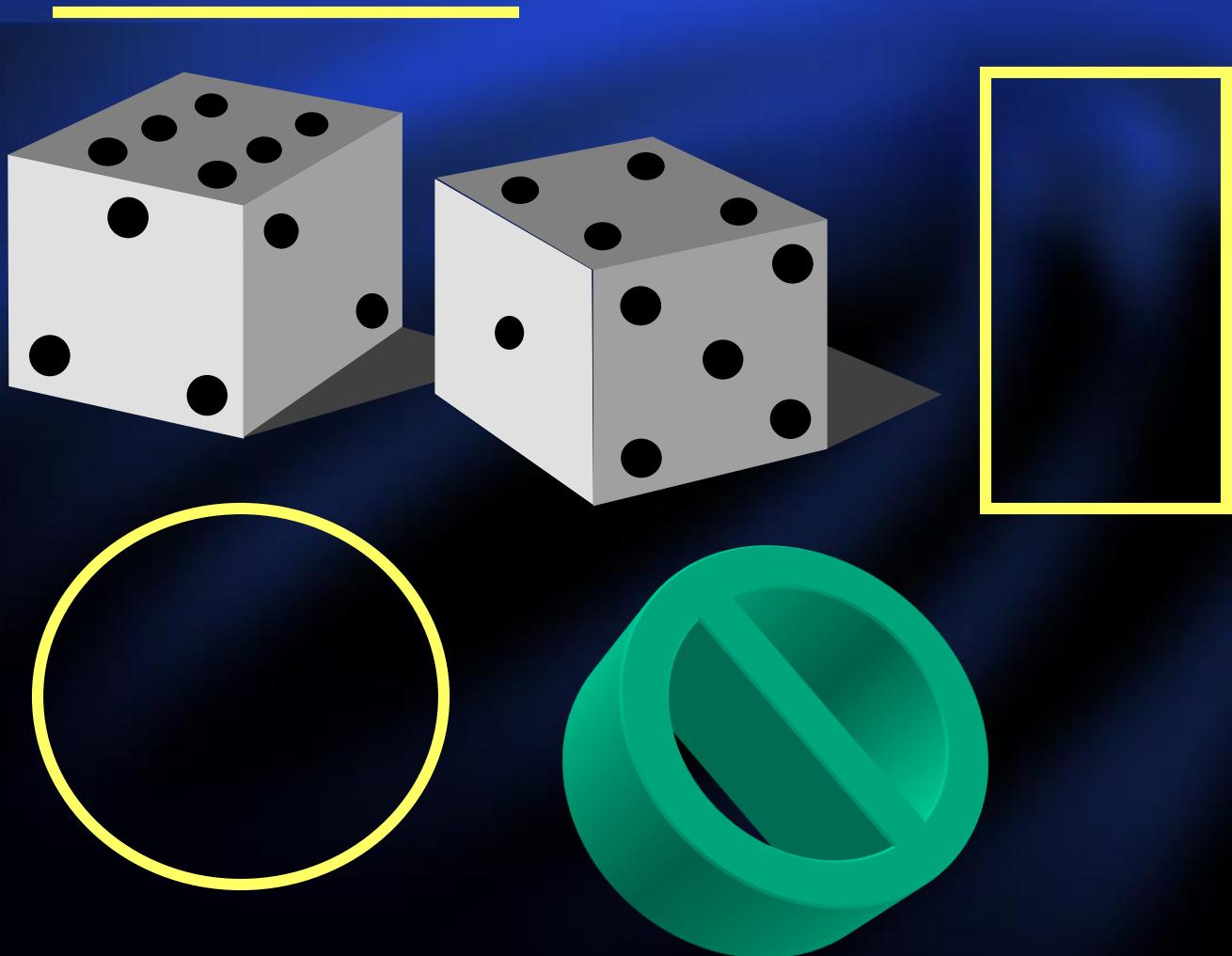


Euclidean Geometry

- A geometry that has an integer dimension
- Sebuah geometri dengan dimensi berupa bilangan bulat:
 - Titik: dimensi = 0
 - Garis,: dimensi = 1
 - Lingkaran: dimensi = 2
 - Kubus: dimensi = 3



Euclidean Geometries



Berapakah dimensi benda-benda yang terlihat pada gambar di bawah ini:

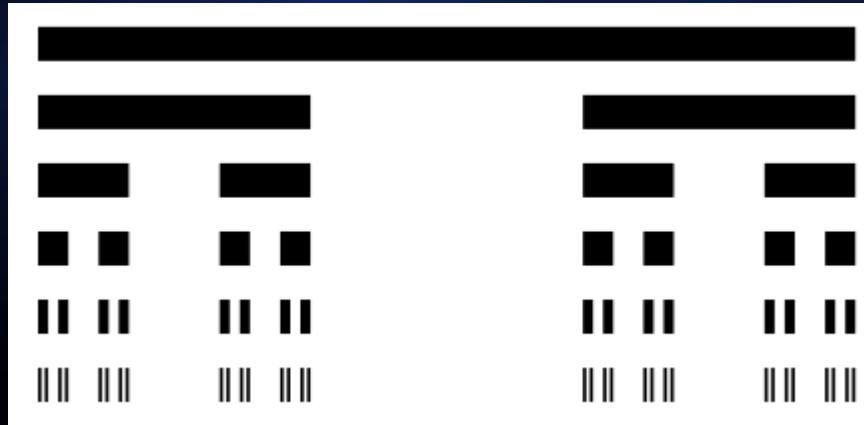


Fractal Geometry:

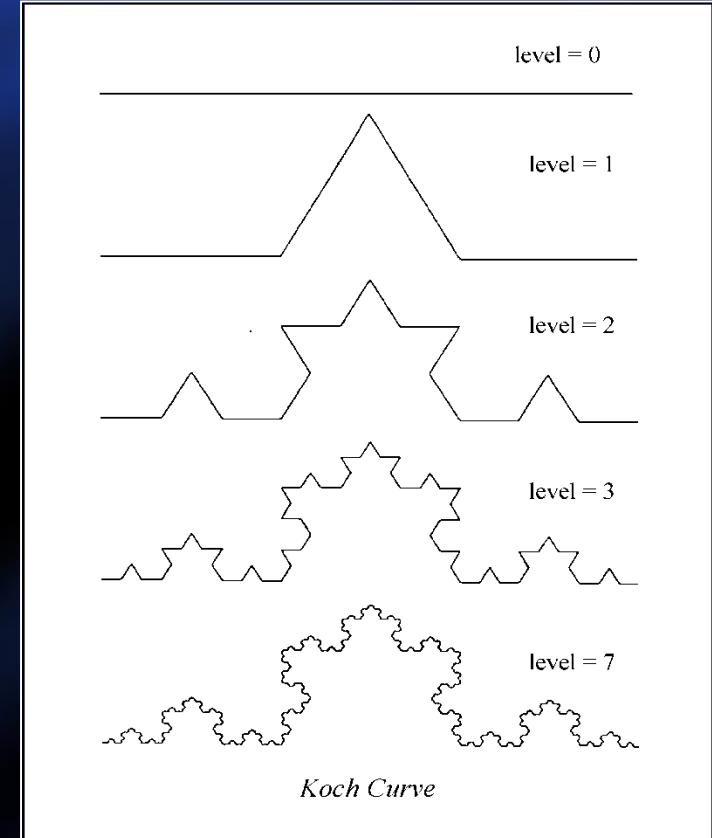
- A set for which the Hausdorff Besicovitch dimension strictly exceeds the topological dimension
- Every set with a noninteger D (dimension)
 - Koch curve, $D = 1.2618$
 - Cantor set, $D = 0.6309$



Self similar geometri fraktal



Cantor Set



Koch Curve

Jagad Raya: termasuk obyek fraktal





Fraktal dí sekítar kíta

Fraktal Brokoli



Awan

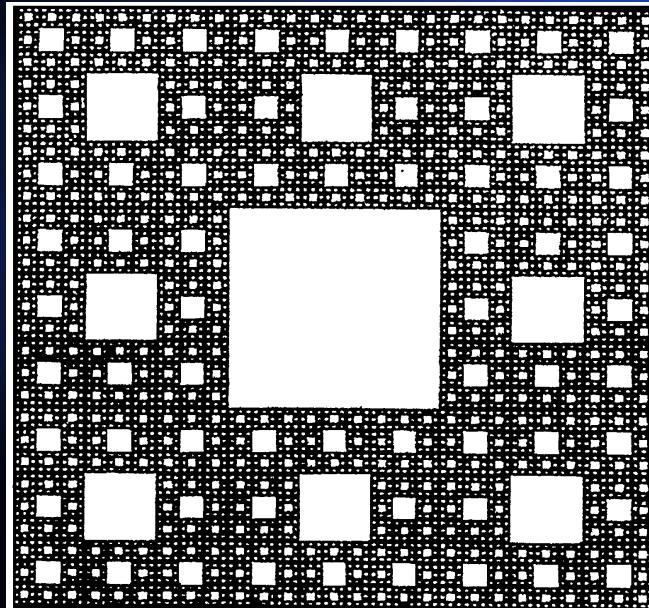


FRACTAL CHARACTERISTICS

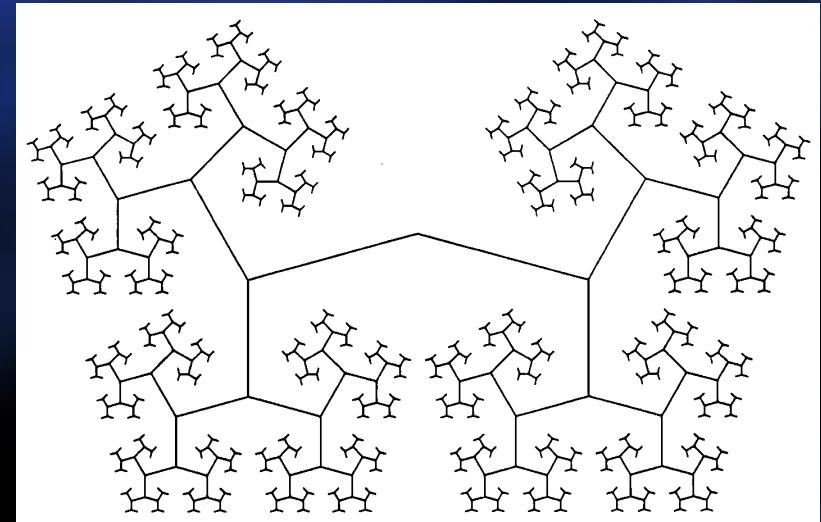
- Self- similarity: serba serupa diri, artinya setiap bagian kecil dalam sebuah fraktal dapat dipandang sebagai replika skala kecil dari bentuk keseluruhannya
- Self- affinity: serba bergabung/berangkai, setiap bagian saling bergabung satu dengan lainnya membentuk keseluruhan fraktal.
- Scale invariant: tidak memiliki skala panjang yang tetap



Self Similarity, Self Affinity



Sierpinski Carpet



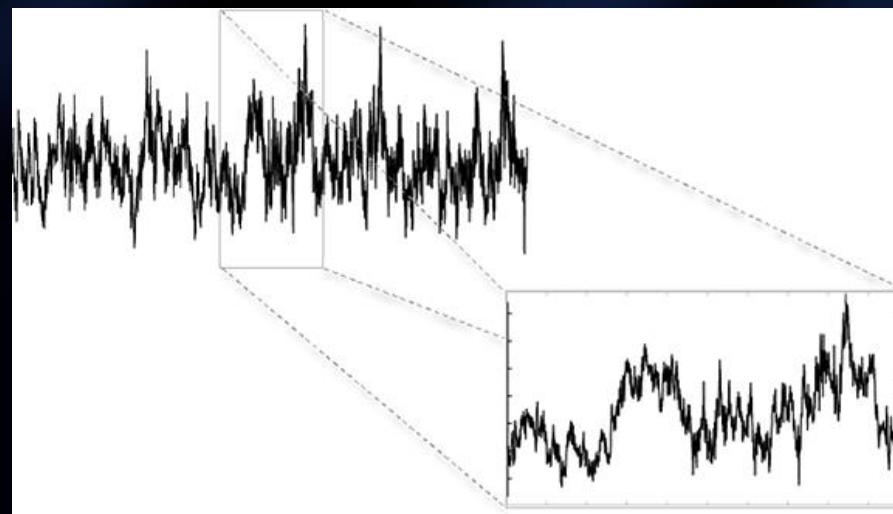
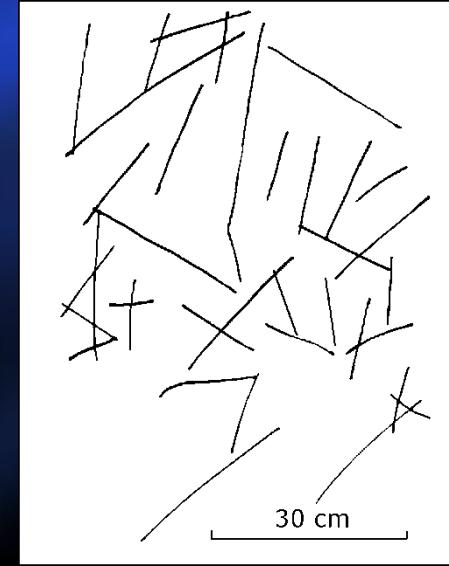
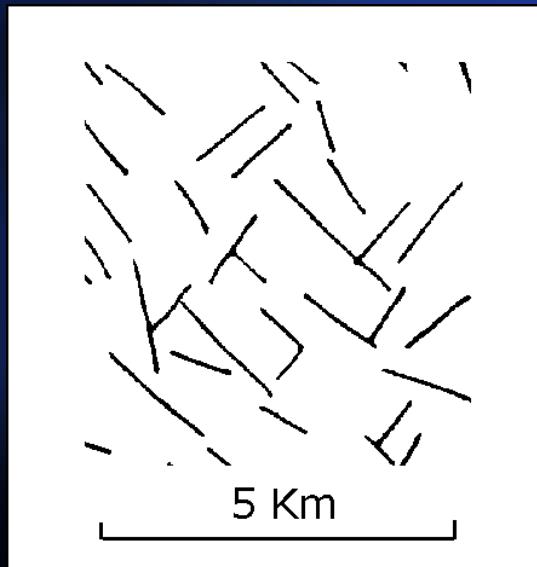
Trees

Self-squared Fractal: Dragon



Self-inverse Fractal: Julia Set

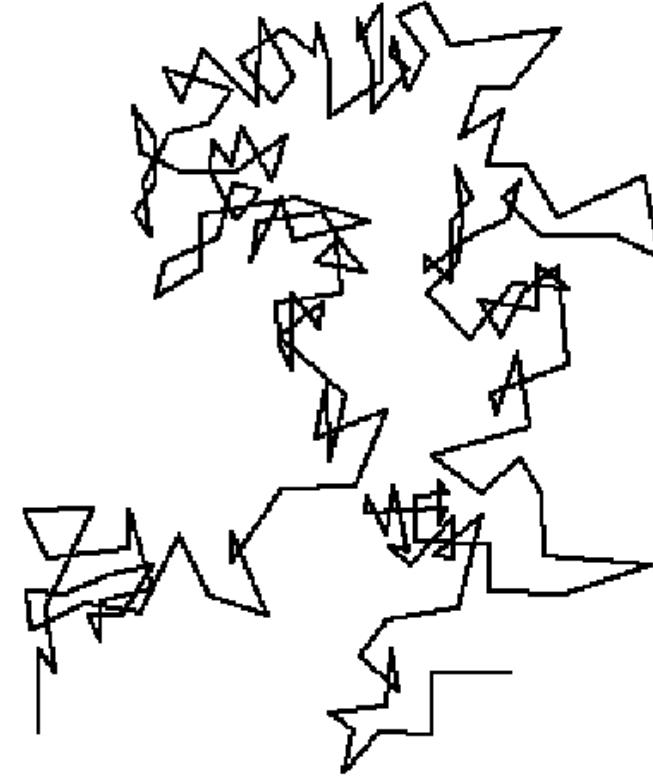
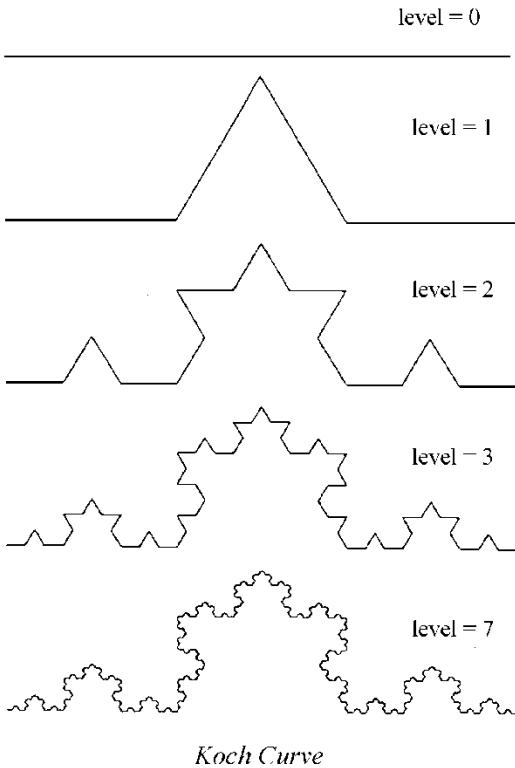
Scale Invariant: Tidak memiliki skala panjang yang spesifik



Types of Fractal:

- Self Similar: serupa diri
- Self Affine: penggabungan diri
- Self Inverse: pencerminan diri
- Brownian: Statistical 2-dimensional fractal
- Gaussian: Statistical 3-dimensional fractal
- Complex (Chaos): Serba tidak beraturan





Self similar:
serupa diri

Chaos: tidak beraturan

Self Similar
Fractal



Self Inverse Fractal: Butterfly



Menger Sponge: self similar - affine fractal



Multí Fractal



Bagian terpenting dalam mempelajari fraktal adalah menentukan harga dimensinya.



Menentukan Harga Dimensi Fraktal

- Metode similarity
- Metode box counting
- Metode balls covering
- Metode Cantor dust

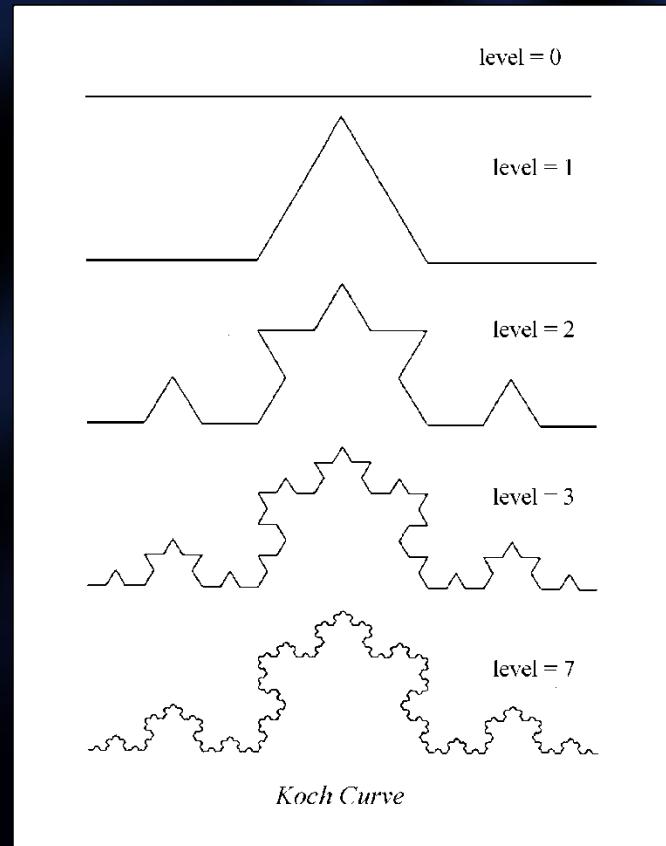


METODE SIMILARITY

Harga dimensi yang diperoleh disebut:
Similarity Dimension

$$D = -\frac{\log N}{\log r}$$

N = jumlah segmen yang bentuknya sama,
r = faktor skala segmen terhadap panjang total obyek fraktal



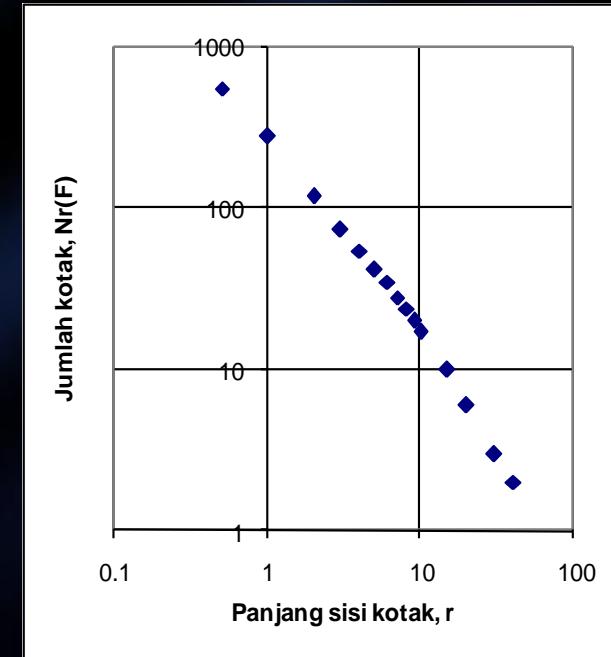
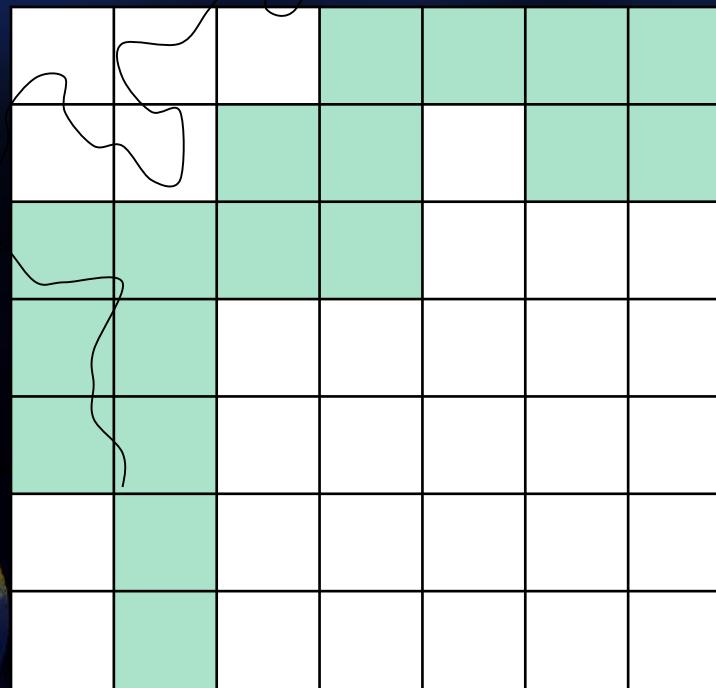
METODE BOX COUNTING

Harga dimensi fraktal yang diperoleh
disebut: **Box Dimension**

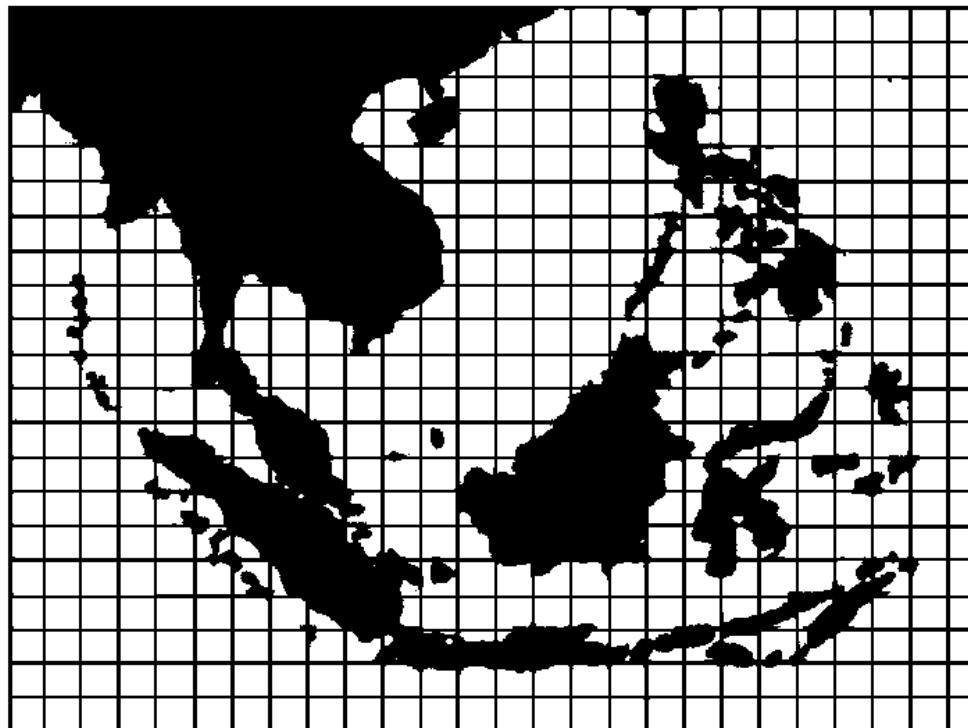
$$D = \lim_{r \rightarrow 0} \frac{\log Nr(F)}{-\log r}$$

r = panjang sisi kotak
Nr(F) = Jumlah kotak yang
melingkupi obyek fraktal

Box counting method: Buat grid melingkupi seluruh obyek fractal. Selanjutnya panjang sisi grid (r) secara berangsur diperkecil hingga mendekati 0, dan diplot ke dalam grafik $x - y$ dg skala log-log



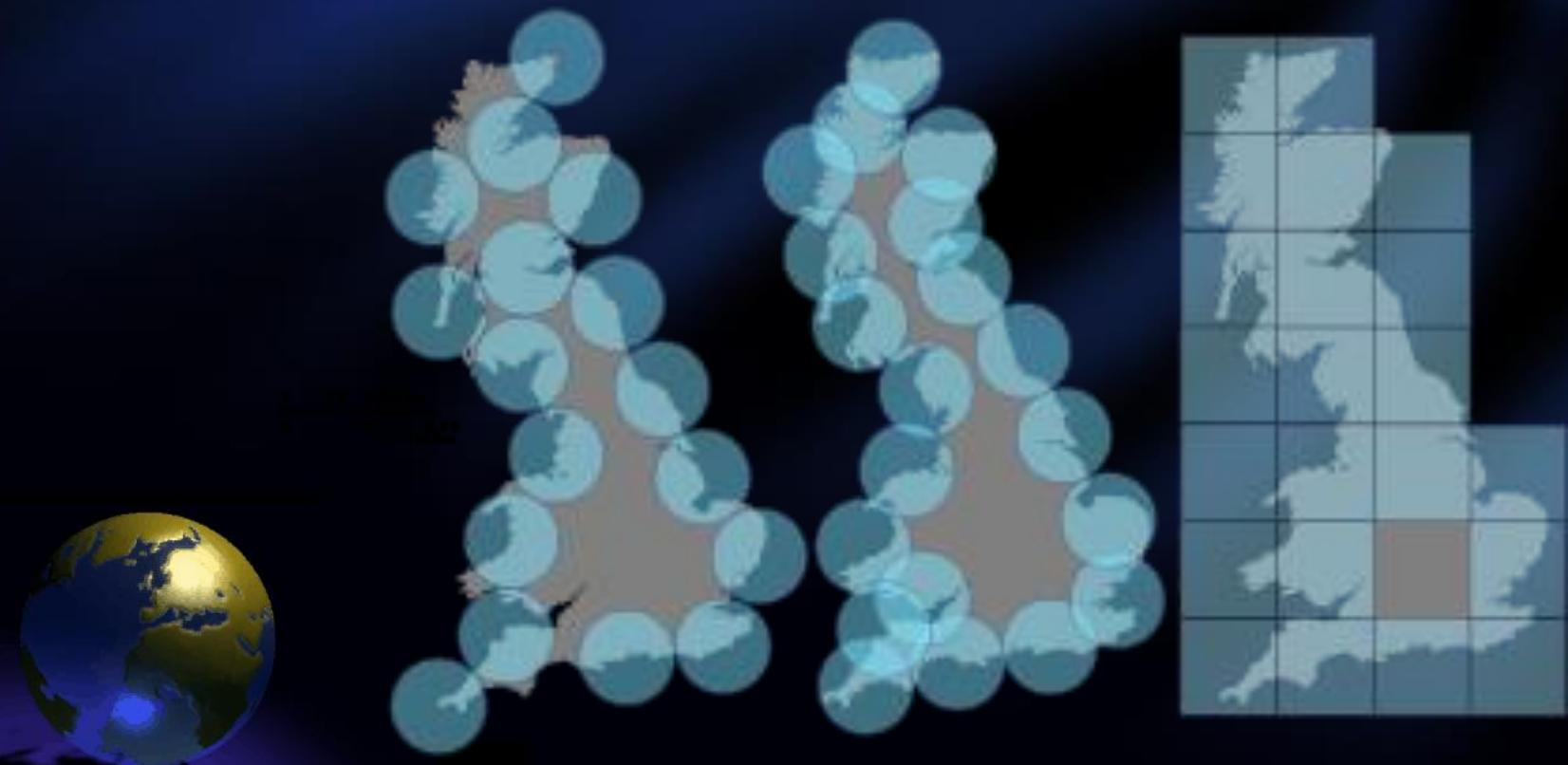
Contoh penentuan harga dimensi fraktal
kepulauan Indonesia, menggunakan
metode box counting



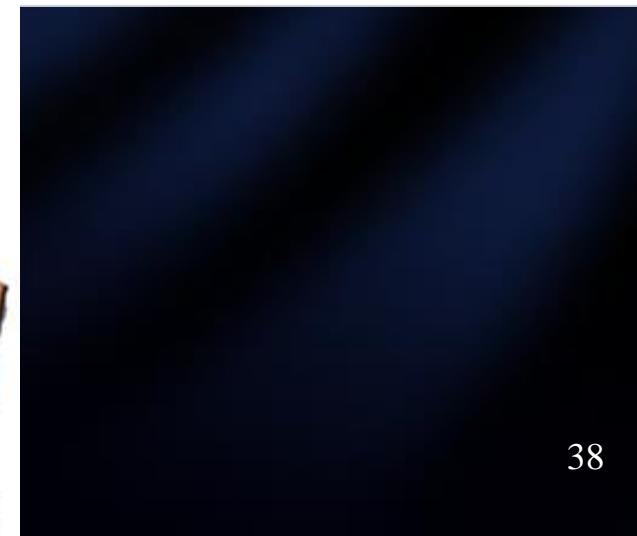
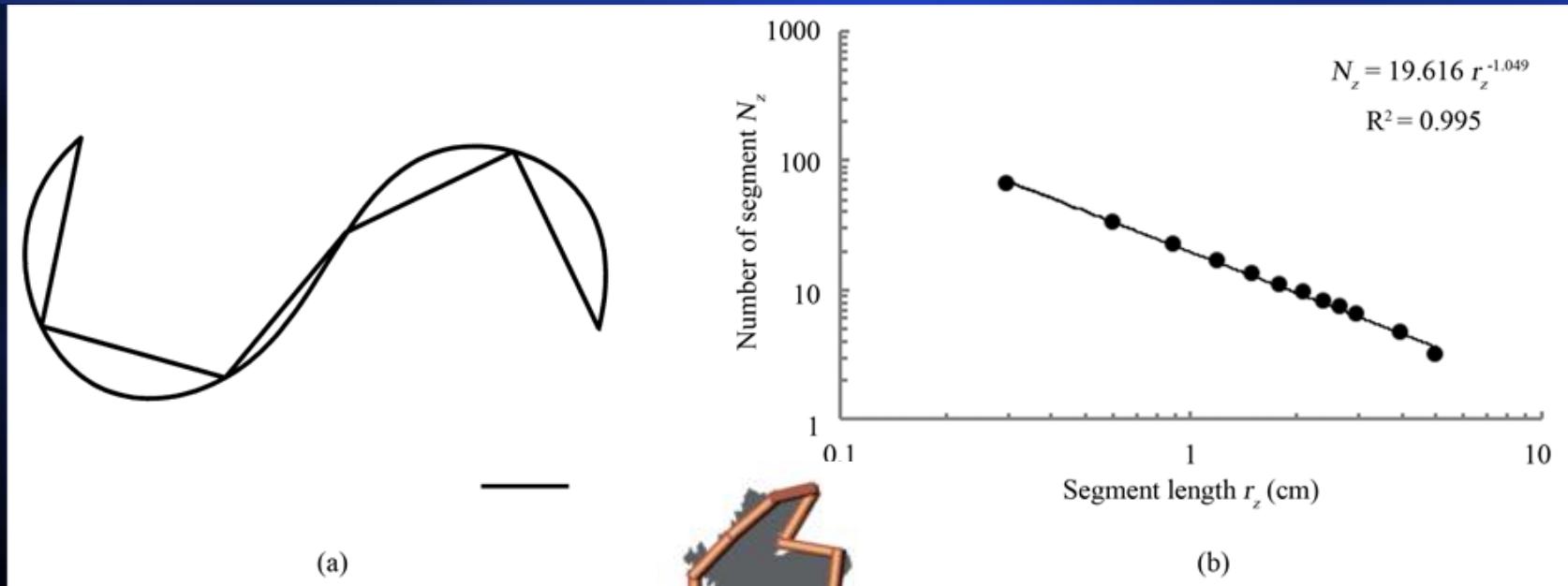
Great Britain: Box counting method



Balls covering method: caranya hampir sama dg box counting method, hanya r bukan merupakan panjang sisi grid, melainkan jari-jari lingkaran yang digunakan untuk melingkupi obyek fraktal



Curve dimension determination dengan metode segmentasi, $r =$ panjang segmen



Penentuan Harga Dimensi Fraktal untuk obyek 2 dan 3 dimensional

Untuk obyek fraktal 2 dimensional: Misal kurva tertutup

$$D = \lim_{r \rightarrow 0} 1 + \frac{\log Nr(F)}{-\log r}$$

Untuk obyek fraktal 3 dimensional:

$$D = \lim_{r \rightarrow 0} 2 + \frac{\log Nr(F)}{-\log r}$$



Cantor Dust Method

- The Cantor's Dust method of fractal analysis has been used to describe the regularity of fracture patterns in geological materials seen on a two-dimensional surface.
- This method is well adapted to a determination of two-dimensional phenomena. The determination of the fractal dimension, D , indicates that it varies regularly as a function of the orientation of the analysis direction.
- The analysis method can also be used to identify local variations in the stress field which caused failure, such as the fractures in a feldspar compared to those in the host granite, etc. The method appears to be a sensitive descriptive measure of the failure patterns in rocks.



Metode penentuan harga dimensi fraktal yang paling sering digunakan adalah box counting, karena relatif mudah dan fleksibel



Kepulauan Cyclades, Yunani: Gaussian Fractal (Dok: Sari BK)



Garis Pantai Selatan Gunungkidul

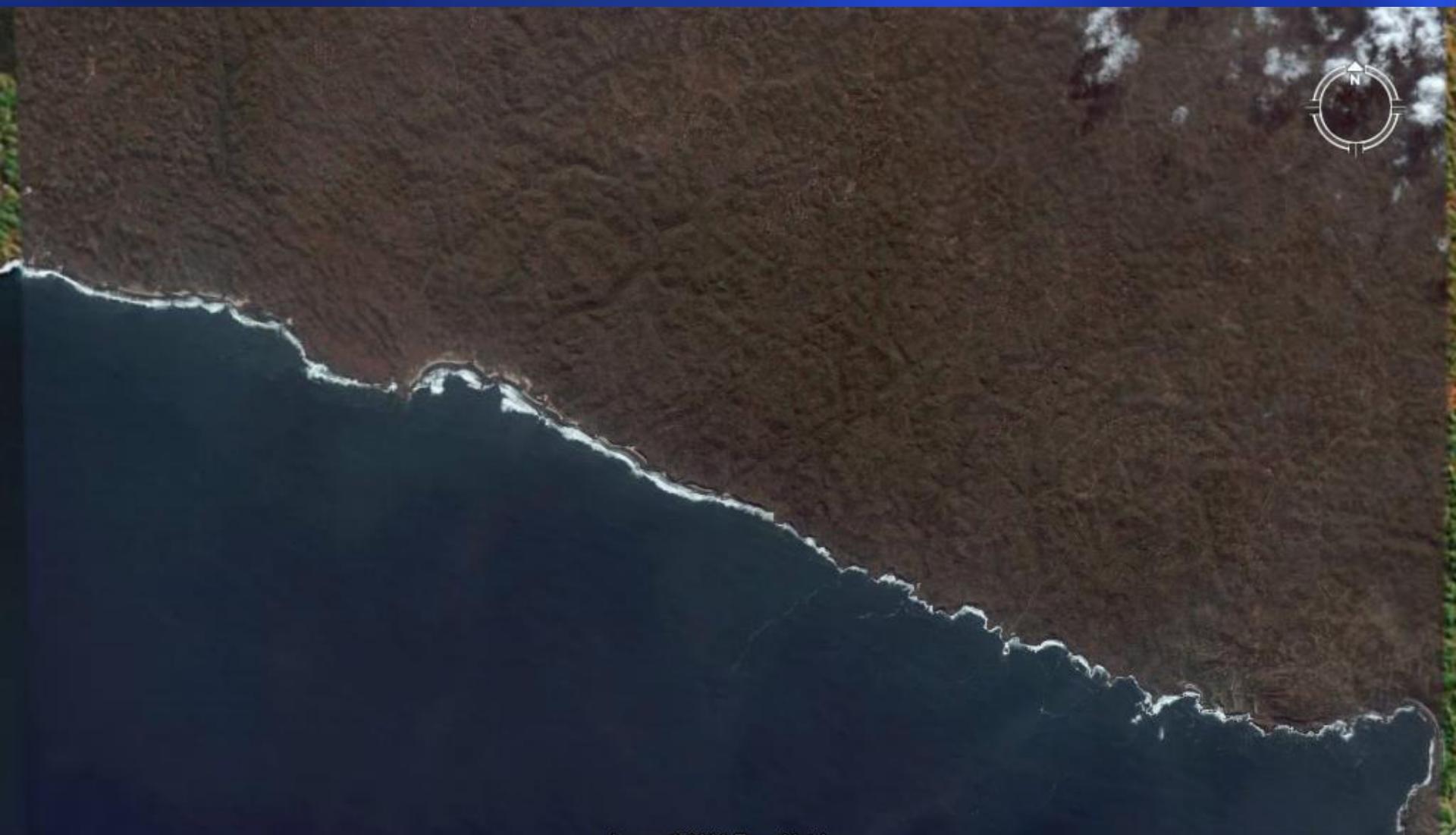


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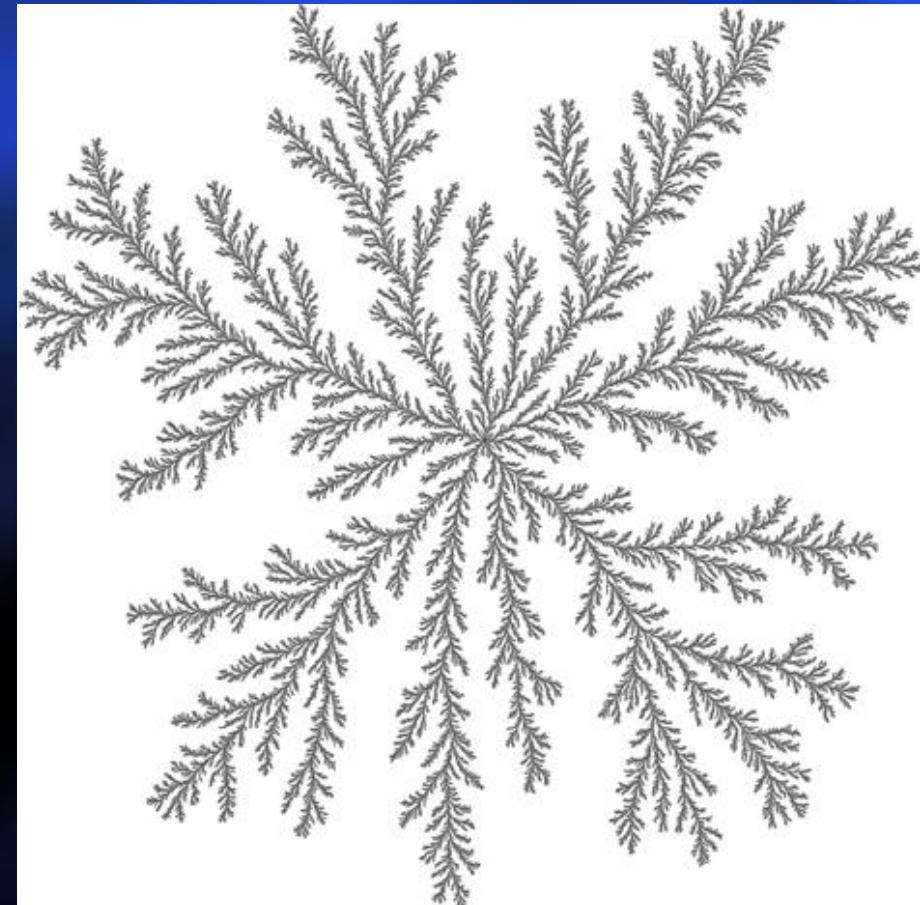
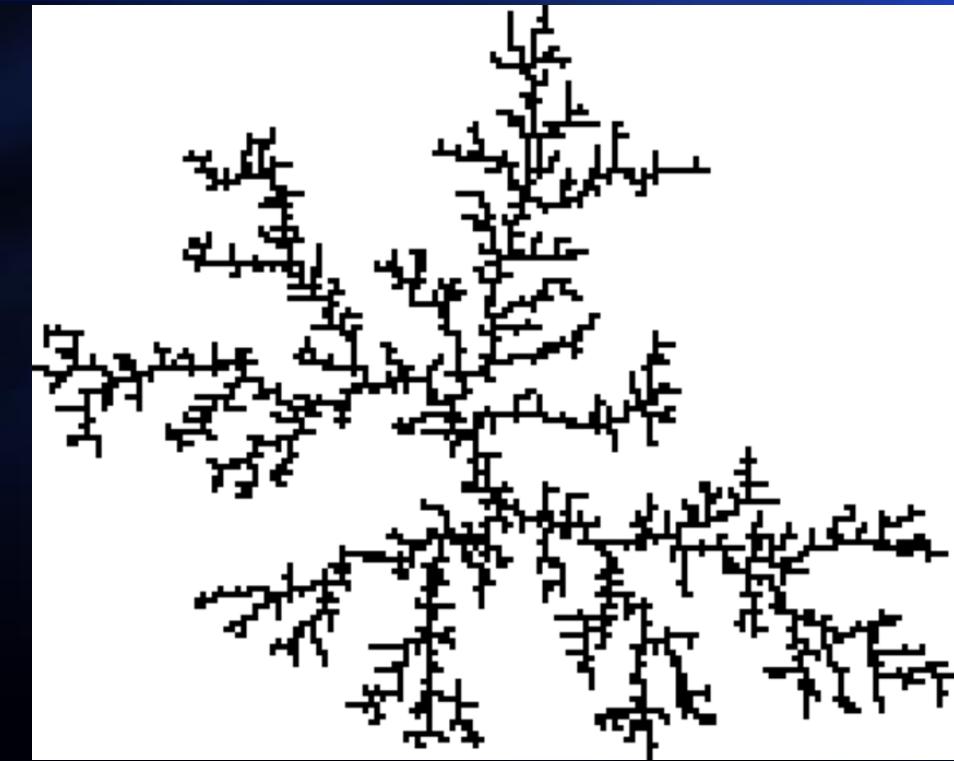
Ombak: Gaussian Fractal
(Dok: Sari BK)

Singkapan Batugamping: Gaussian Fractal (Dok: Sari BK)



“Wedhus Gembel”, termasuk tipe fractal yang mana? (Dok: Sari BK)





Diffusion Limited Aggregation (DLA)

Pola Garis Kontur Topografi Karst

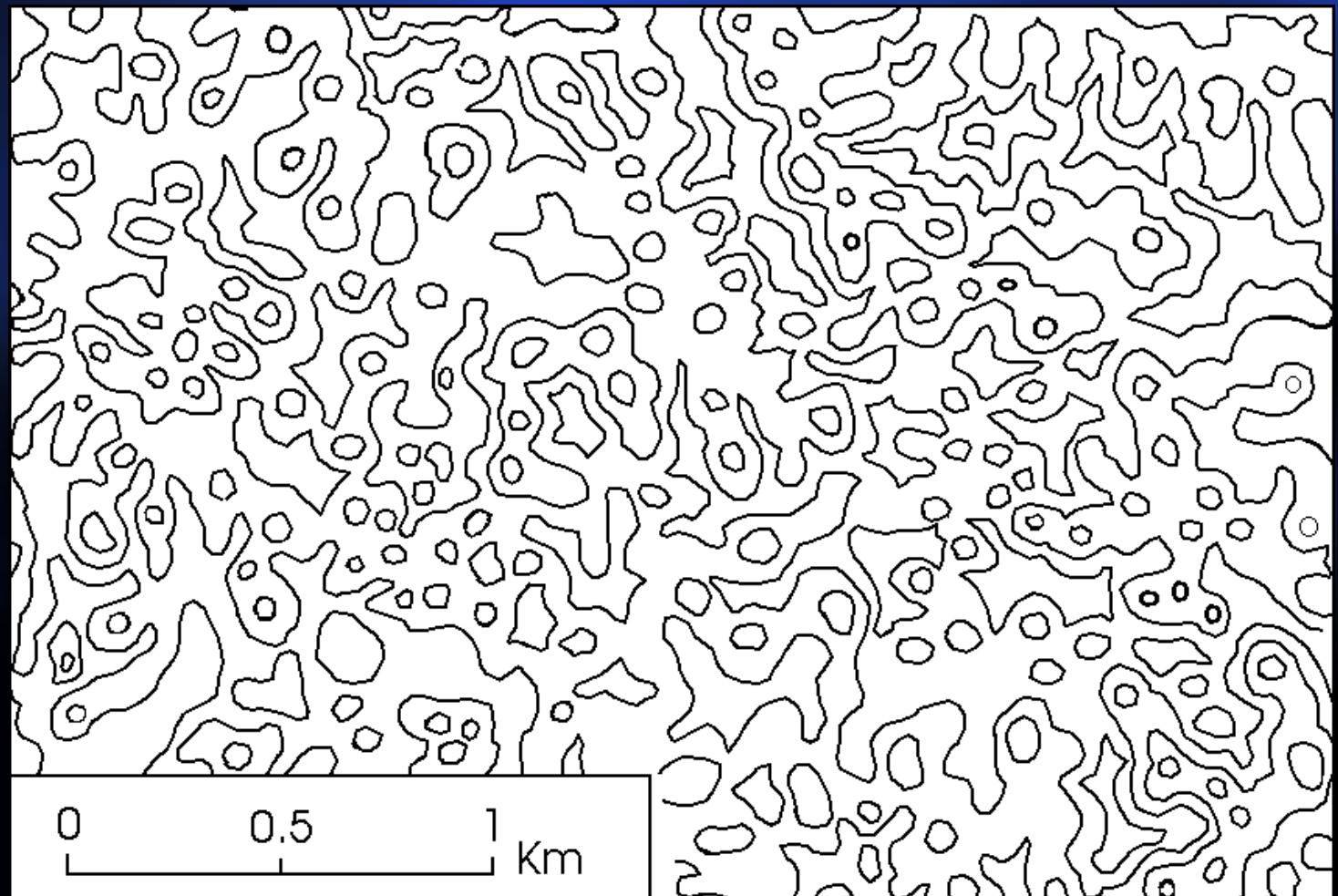
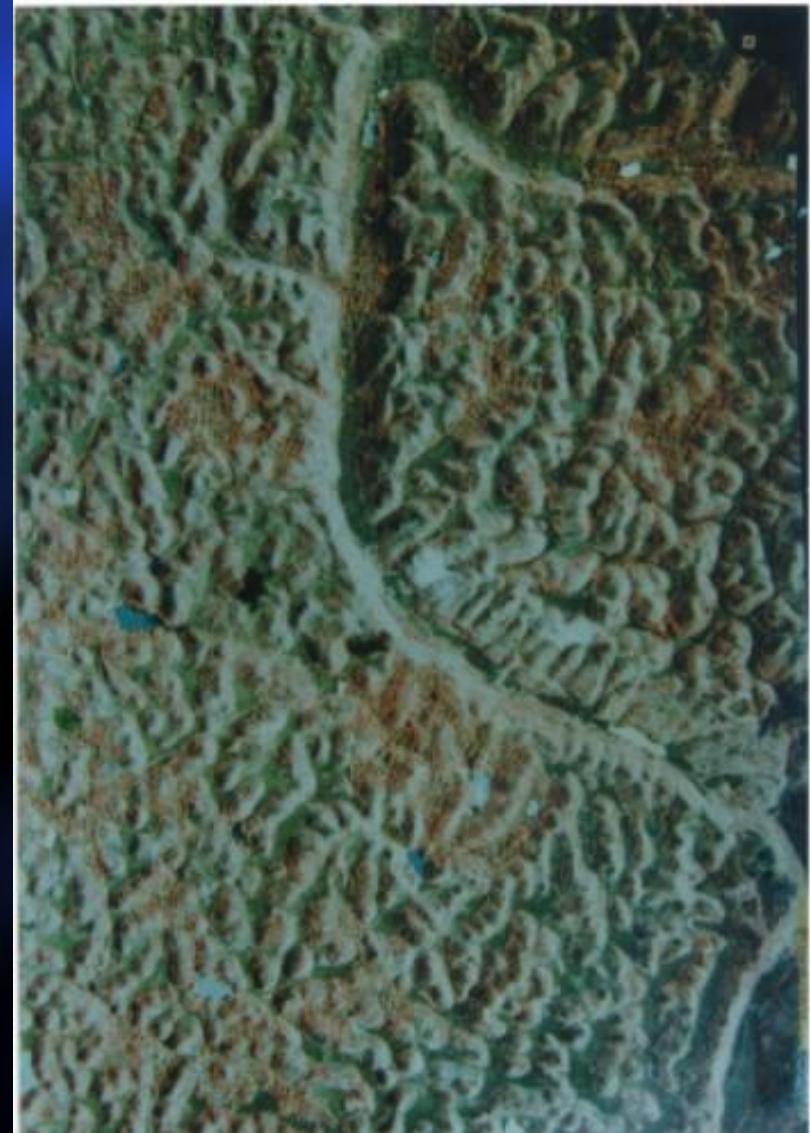
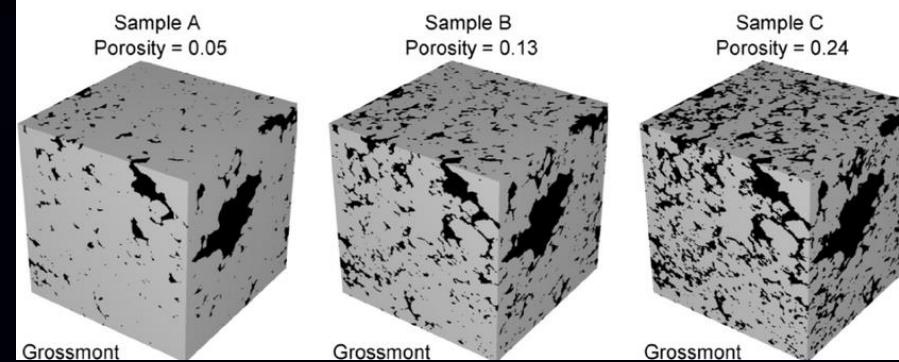
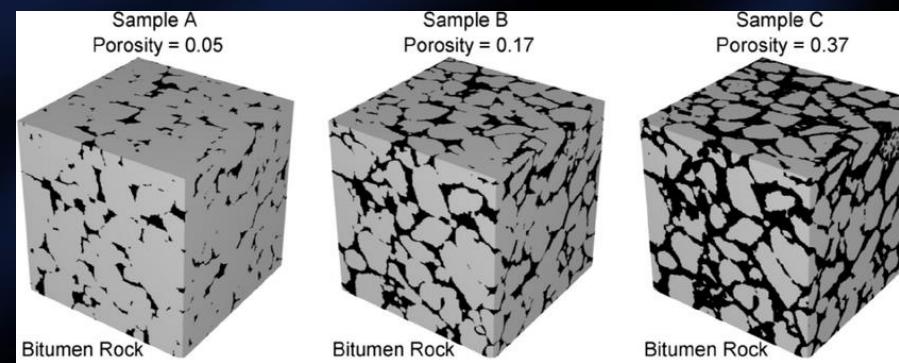
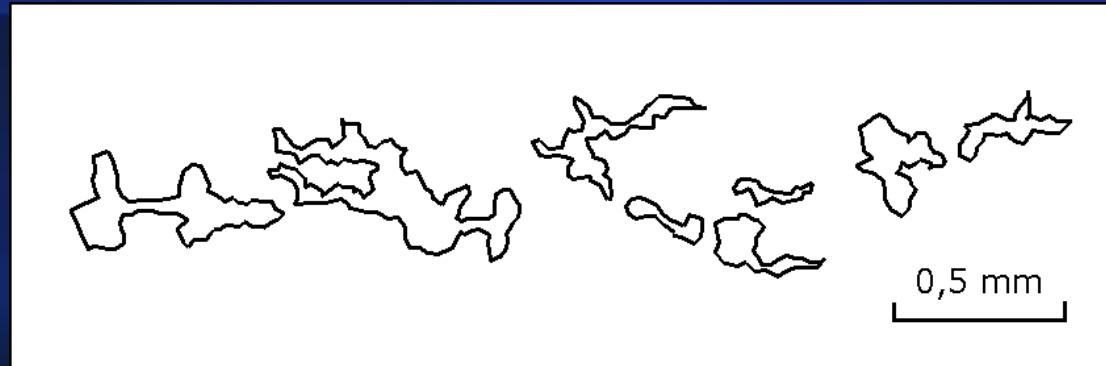


Foto udara
Lembah
Bengawan Solo
Purba



Penampang Porositas Batuan



*Sekian
Terima kasih*

